ACADÉMIE DES SCIENCES SOCIALES ET POLITIQUES
DE LA RÉPUBLIQUE SOCIALISTE DE ROUMANIE

REVUE ROUMAINE DES SCIENCES SOCIALES
SÉRIE DE PHILOSOPHIE ET LOGIQUE

TIRAGE À PART

TOME 28 Juillet — Décembre N^os 3 — 4, 1984

EDITURA ACADEMIEI REPUBLICII SOCIALISTE ROMÂNIA

MODELS AND THEORETIZATION

ADRIAN MIROIU

In a previous paper (Miroiu 1984a) I used modal semantics in an attempt to reconstruct J. D. Sneed's concept of a 'theoretical function'. The approach also seemed to have a certain relevance upon the logical structure of two intertheoretical relations: specialization and theoretization and I believe that some fair grounds were advanced to hold that they are closely interconnected. It looks to me that there it is not possible to explain theoretization without any recourse to specialization, for, indeed, theoretization, if accurately taken into account, needs be described by appeal to some sorts of propositions one would naturally be ready to identify with special laws or constraints (here by proposition I mean a logical construct: that which is expressed by a sentence of the theory).

Let T be a physical theory and let $^\wedge A$ be a T-proposition. Logicians do usually identify propositions with sets of possible worlds: a world w is in $^\wedge A$ when $^\wedge A$ is true at w. Note that constructing propositions needs not a syntax; propositions are not linguistic, rather they are conceptual entities. When a theory T is concerned, propositions are usually understood as regions in T's phase state and $^\wedge A$ contains a state iff the relations expressed by A hold at it. However, within Sneed's formalism, it seems — for reasons I cannot discuss at length here — that a better choice is this: a T-proposition would be identified with a set not of models, but of *potential models* of T. Hence, one can give a better account of T's logical and empirical claims; on the other hand, constructing a T-proposition requires the notions of T-potential model and T-model: but, as Sneed argues, they involve the whole of T's mathematical structure.

Now, some problems arise concerning the notion of *law for a theory* T (a T-law). First, a T-law is not necessarily a maximal proposition: $^\wedge A$ is a T-law when the set of all T-models (M) is in $^\wedge A$ and thus some of the T-potential models (= some Mp's) are not required to belong to $^\wedge A$. Then, which is the relation laws bear to *constraints*? Constraints, on Sneed's account, express cross-connexions, i.e. a transport of information, among different potential models and are formally identified with n-ary relationships on the set Mp (in a particular case, a constraint is a binary relationship on Mp, i.e. a set of pairs of elements of Mp). Sneed does not appeal to constraints C when defining T-laws; in fact it is not possible, for — as easily seen — laws and constraints are formally heterogeneous. This is the reason why I introduced the concept of *generalized law* (Miroiu 1984a). A generalized law is an n-ary relation on Mp and it could be conceived of as a proposition in the sense of n-dimensional semantics — or, more generally, in the sense of local semantics (Miroiu 1984). Standard

Rev. Roum. Sci.Sociales — Philosophie et Logique, 28, 3—4, p. 273—330, Bucarest, 1984.

laws (laws on Sneed's approach) are naturally translatable into generalized ones.

The important point I like to mention is this: one can define now T-laws so that constraints would be essentially involved. I used the following example. Let $\hat{} A$ be a generalized law and let $\hat{} C$ be a constraint (=a binary relation on Mp). The corresponding $\hat{} CA$-law (actually $\hat{} CA$ is a conditional $\hat{} (C \rightarrow A)$ proposition) is constructed so that relation

1. (i, i') is in $\hat{} CA$ iff for all j, if (i', j) is in $\hat{} C$, then (i, j) is in $\hat{} A$.

would hold. Suppose $\hat{} A$ is a standard law. Then:

2. i is in $\hat{} CA$ iff for all j, if (i, j) is in $\hat{} C$, then j is in $\hat{} A$. Hence if $\hat{} A$ is a standard law, $\hat{} CA$ is a standard law, too. But, if one starts with $\hat{} A$-laws and $\hat{} CA$-laws, constraint $\hat{} C$ is definable by:

3. (i, j) is in $\hat{} C$ iff for all $\hat{} A$, if i is in $\hat{} CA$, then j is in $\hat{} A$.

Consider, e.g., a standard law $\hat{} CA$. Each standard law is valued at a point i in Mp and we say that $\hat{} CA$ is true at i exactly when i is in $\hat{} CA$. But, it is important to note that $\hat{} CA$ conveys some information (*via* constraint $\hat{} C$) about other potential models $j, j' \ldots$ in Mp. I claimed that these generalized laws play an essential function in theoretization contexts. Indeed, it is possible to argue that given a theory T', constraints on its non-theoretical functions could not be identified with cross-connexions on its potential models; nevertheless, if T' is a theoretization of T, and if $\hat{} C$ is a constraint on a T-theoretical function (i.e. $\hat{} C$ is definable as a relation on Mp), then T' somehow involves $\hat{} C$. Which are the mechanisms that make it possible? I believe that a satisfactory answer is this: T' contains T-laws, but they could not be treated as cross-connexions on T''s potential models; rather they are confined inside T'-potential models. If the T-law is reducible to a standard $\hat{} CA$-law, the answer is obvious.

However, in Miroiu (1984a) I left unsolved the general case. How is there possible to manage at T' a non-reducible generalized T-law? A promising path I suggested is to exploit some of Segerberg's concepts (Segerberg 1973) [1]. It is this issue I shall be concerned with in this paper.

Modal semantics provides us with conceptual means I take to be workable when investigating the logical structure of the family of T-propositions. It is this point that has to be vigorously stated, for the approach is interesting, in my view, not because of its technical, but *theoretical* and *methodological* advantages. Moreover, I believe that in this case technical sophistication is not necessary, methodologically unimportant and sometimes counterintuitive. Local semantics, as described in Miroiu (1984), e.g avoids such non-necessary sophistication while asking for some different (but intuitive — were we ready to forget much of our philosophical *apriori*) basic assumptions[2]; see, in this respect, the local semantics approach to conditionals.

A standard law $\hat{} A$ is valued at a certain point i (a possible world = a potential model of T) and we say that $\hat{} A$ is true or not at i. But, to value a constraint $\hat{} C$, a pair (i, j) of points is needed: $\hat{} C$ is true or false at i with respect to j, or, to put it in other words, $\hat{} C$ is true or false at (i, j).

[1] It is not difficult in turn to reconstruct Segerberg's results within the frame of local semantics (Miroiu, 1984). I still prefer to use his bi-dimensional semantics simply because it seems to rest on somehow more familiar grounds.

[2] The arguments I presented in Miroiu (1984) towards a local theory of rigidity are heuristical in nature. From a logical point of view, the case is simpler, for the puzzle roughly falls over the requirement that world-variables be rigid.

Now the basic (Segerbergian) claim is this: the phrase 'proposition $^\wedge A$ is true at the pair (i, j) of points' is to be restated as: '$^\wedge A$ is true at a point g' (which is endowed with an inner structure). This statement is, at first, puzzling, for the switch seems to be only a verbal one. Indeed, we have no guarantee that g is one of the valuation points i, j, j'...; second, if a pair (i, j) of T-potential models were identified with a point g, nothing seems to have been gained. What is meant then is this. The notion of 'valuation point' is ambiguous. Roughly speaking, a valuation point can be specified either as a pair of potential models, or as a potential model (perhaps with some additional special properties).

I agree that such a specification raises great difficulties. However, it seems to me that some hope still comes from Segerberg's work. Let me turn to his notion of *diagonal* and related concepts. Each possible world w, Segerberg argues, could be constructed as a pair (w', w'') of worlds. If w is to be identified with a pair (w', w'), say that w is on the diagonal. Now, given a world w, then there must be two worlds w' and w'' on the diagonal so that $w = (w', w'')$. The problem could be treated in a more general manner as follows: we have, as concerns a possible world, two irreducible sorts of information; in this sense, the world is to be specified by two coordinates — its longitude X and its latitude Y. Of course, it is natural to assume that two worlds are identical exactly when they coincide on both coordinates. (The logician, who is mainly interested in abstract structures, wishes to explore other (more or less intuitive) conditions, e.g.: for each possible world (w', w'') do exist: 1. a world (w', w'); 2. a world (w'', w''); 3. a world (w'', w') a.s.o.)

Now I return to theory T. It comes possible to represent each T-potential model i as a pair (i', i''), where i' and i'' are, formally, Mp-type entities. Let DMp (the diagonal of Mp) be the set of all these entities. Obviously, the first question that comes to mind is this: is DMp a subset of Mp? To answer this question, I think it is profitable to take into account a particular physical theory and let it be CPM (classical particle mechanics). There are two CPM-theoretical functions: force f and mass m, but, for the sake of simplicity, I shall consider only function m. Sneed proves that m's being a CPM-theoretical function amounts to: any description of a method to determine the mass $m(x)$ of an object x appearing in a certain application i of CPM presupposes that there is (another) successful application j of CPM. I reconstructed this definition in Miroiu (1984a) in a modal jargon. The central idea looks to be the following: let i be in Mp and let Xi be the set of all objects appearing in i. The potential model i could be used so as to determine at a certain model j the mass of objects (not necessarily appearing in i) in a set Yi. Now the claim that m is a CPM-theoretical function can be restated as: $^\wedge(m(x) = k)$ is true at i if 1. x is in Xi; there is a potential model j so that: (a) $C(i, j)$; (b) x is in Yj and by j $m(x) = k$. For each potential model i we have (as regards function m!) two kinds of information, an X-information, concerning the set of objects existing in i, and an Y-information, concerning the set of objects with mass fixed by appeal to i [3].

[3] Actually, the argument is more complicated, for first i fixes the mass of x with respect to some j; second, i shall be a successful application! I find there are two possible approaches to this puzzle: to assume that what is concerned is a *normal* understanding of the theoretical (Miroiu 1984a); or to argue that the notion of successful application is not context-free. I believe this second view is more promising: being successful is not an absolute, but a contextual predicate. The full phrase is then this: 'i is successful relative to j'.

A potential model i is then identified with a pair (Xi, Yi). The important point Sneed succeeded to prove is that in general $Xi \neq Yi$. But if i is not able to fix the mass of all the objects appearing at it, it is necessary to assume a transport of information to i from other potential models to do that. Mass is independent of system, Sneed claims, i.e. if x is in Xi and x is in Xj, then i is in $\hat{}(m(x) = k)$ iff j is in $\hat{}(m(x) = k)$ too.

It is then possible to construct this constraint as follows. First assume that:

4. If by i, $m(x) = k$ and x is in $Xi \cap Yi$, then i is in $\hat{}(m(x) = k)$. In other words, if a potential model is used to determine the mass of an object x, then if x appears in i, it is i and not another potential model which fixes its mass. Let i and j be two Mp's. Define $i' = (Xi', Yi')$ and $j' = (Xj', Yj')$, with $Xi = Xi'$, $Yi' = Yj$, $Xj' = Yj$, $Yj' = Yi$. Assume that:

5. Potential model i' exists iff j' exists too. Now it is possible to define constraint $\hat{}C$ (independence of system) on function m:

6. $C(i, j)$ iff for all x, if x is in $Yi \cap Xj$, then j is in $\hat{}(m(x) = k)$ iff by i, $m(x) = k$.

Supose that for some $i = (Xi, Yi)$ it holds that $Xi = Yi$. In that case, say that i is on the diagonal DMp of Mp. Suppose further that

7. If there is a potential model $i = (Xi, Yi)$, then there are two potential models $i' = (Xi, Yi')$ and $i'' = (Xi'', Yi'')$, with $Xi = Xi' = Yi'$, $Yi = Xi'' = Yi''$.

(7) states that the diagonal DMp exists and that it is able to mirror Mp, i.e. it is possible to reconstruct each element i of Mp as a pair (i', i'') of elements of DMp. DMp is assumed to be a proper subset of Mp, but it conveys the whole of the information we have about Mp. But there is another problem: is (7) consistent with Sneed's philosophy of physics? Why, for each i would there necessarily exist a potential model i' on the diagonal? For a moment I leave this question unanswered, for it is necessary to make a short comparison with non-Sneedian philosophies of physics.

Non-Sneedians agree that for each $i = (Xi, Yi)$, $Xi = Yi$. Of course, some of the consequences of this view are puzzling and create problems of their own. How is there possible, for instance, to show that two rods are congruent? This relation must be, according to H. Reichenbach, either an empirical discovery, or postulated by convention. But, while the later alternative is dubious, the former brings about circularity. W. Stegmüller (1979, p. 19) offers a very good description of the situation from a Sneedian perspective. His argument, roughly speaking, is this. The troubles about circularity are ruled out once it is recognized that a theoretical concept is involved. We are forced to pass from one to another application of the theory whenever we deal with a theoretical magnitude: that is the sense of Sneed's idea that in general $Xi \neq Yi$.

These show that while non-Sneedians take *all* (potential) models lie on DMp, Sneed states that some potential models are not on the diagonal. However, the claim that some potential models are not on the diagonal means that: 1. there must be some cross-connexions among potential models; 2. among those connexions, *one is privileged*. Sneed devoted his entire attention to the first of the two aspects and created the notion of *constraint* as a means to describe the way in which a transport of information among potential models holds. But the second aspect is, I believe, equally important, though Sneed does not explore its implications. The case is this : some models are not on the diagonal, but non-Sneedian philosophy of science was *concerned only with diagonal models* ! What Sneed is missing, I think, is an *appropriate reconstruction* of the notion of diagonal. (Note that this does not entail that (7) holds, i.e. that DMp is a subset of Mp; but it entails that logical mechanisms to reconstruct DMp are needed.)

My basic claim is that the notion of (potential) model is itself context-dependent. It is no absolute sense in which one could say that i is a potential model (as Sneed and Stegmüller assume). Up to this very moment, models were involved : 1. as logical constructs (entities of which it makes sense to ask if T's mathematical structure is satisfied) ; and 2. in the context of *specialization*. Now I want to suggest that in the context of theoretization the notion of model gains another sense. Let T' be a theoretization of T. T-models are identified with T'-partial potential models. T' presupposes T, therefore it must contain all of its theoretical mechanisms. But T-constraints could not be identified with cross-connexions among T''-potential models — they must be internalized into the very structure of the T'-partial potential models. However, if this is true, then a T-(irreducible) generalized law $\hat{}A$ has to be valued at *a point* i(i is a T-model and a T'-partial potential model). Suppose $\hat{}A$ is a set of pairs of T-potential models. If now we move to T', then it is necessary that $i \in \hat{}A$ would make sense. But that is possible only if i is on the diagonal of Mp, i.e., if $i = (j', j'')$ and $j' = j''$.

Hence, I hold that T-partial potential models are *not* T-models, but elements of DM (= the diagonal of M)! But, according to Sneed, it is debatable if DMp and, consequently, DM shall exist. In fact if relation (7) is obtained, it becomes then possible to rule out both constraints and also each potential model outside DMp.

I do not think (7) is true of all potential models i in Mp. DMp, therefore, does not exist ! but that cannot be true.

The dilemma is easily solved once we admit that being a potential model is a context-dependent notion. DMp is a family of T-potential models *only* in the context of theoretization. DMp could not exist, e.g., when specialization relations are the only concerned ; but from the point of view of T' (conceived of as a theoretization of T), DMp *is* the class of all the potential models of T.

To sum up : specialization determined T-constrained models ; second, theoretization provides T-diagonal models. A third fundamental intertheoretical relation, Sneed holds (Balzer and Sneed, 1977—1978), is reduction. Reduction defines limit models. However, it is not the aim of the present paper to discuss at length that notion ; I should only like to mention that a limit model of T is, in my view, satisfactorily describable in Mayr's formalism (Mayr, 1976—1979) : as a limit of a convergent series of models of T'.

REFERENCES

BALZER, W., SNEED, J. D. (1977—1978), *Generalized Net Structures of Empirical Theories*, in "Studia Logica", 36—37.

MAYR, D. (1976—1979), *Investigations in the Concept of Reduction*, in "Erkenntnis", 10 (1976), 13(1979).

MIROIU, A. (1984), *A programme for Local Semantics*, in „Revue Roumaine des Sciences Sociales, Série de Philosophie et de Logique", 28, 2, p. 146–154.
MIROIU, A. (1984 a), *A Modal Approach to Sneed's Theoretical Functions*, in „Philosophia Naturalis", Band 21, Heft 2.
SEGERBERG, K. (1973), *Two-Domensional Modal Logic*, in "Journal of Philosophical Logic", 2, p. 77–96.
SNEED, J. D. (1971), *The Logical Structure of Mathematical Physics*, D. Reidel Publ. Comp., Dordrecht.
STEGMÜLLER, W. (1979) *The Structuralist View of Theories*, Springer Verlag, Heidelberg.