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A PROGRAMME FOR LOCAL SEMANTICS

BY

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Possible worlds semantics is often regarded as a very useful tool in the hands of logicians, i.e. to construct models of the language that will help to give a systematic overview of the patterns of valid inference. However, some authors thought that possible worlds have certain philosophical importance and used them in the treatment of metaphysical puzzles (the analysis of necessity; the nature of properties, propositions, sets and other "philosophical" entities; the analysis of essentialism; the function of proper names and of descriptions; the role of individuals, a.s.o.), while others denied any philosophical relevance of the approach.

I am a realist about possible worlds (at least in the minimal sense in which it would imply that some objects are not, and some could not exist), but I also disagree with their use in philosophical matters. Let me explain that. I don't think there is any analogy between possible worlds and, say, the little billiard balls the nineteenth-century physicists conceived of as fictional models of gas molecules. A better analogy would be the following: the Bohr-Rutherford theory was a theory of electrons, but the way it

[27] N. Chomsky, *Réflexions sur le langage*, Fr. Maspero, 1977, pp. 12, 13.
[28] H. Marcuse, *L'homme unidimensionnel*, Éd. du Minuit, 1968, pp. 200—211.
[29] E. Gellner, *Words and Things*, Victor Gollancz, Ltd., 1959.
[30] B. Russell, Preface to 29, pp. 14, 15.

described them was not the correct one (at least from the standpoint of present theories). I think that the possible worlds approach is philosophically significant, but I also think that the descriptions we use to pick up possible worlds is mistaken.

Possible worlds are usually identified with maximal states of affairs and I believe that this intuition is correct; but it seems to me that its regimentation into the philosophical parlance is not the best one. Indeed, possible worlds are reconstructed as maximal states of *non-modal* affairs. Let me look around for an example. Think of time as a set of integers under the natural ordering and understand W (A, x) to mean: "The sentence A is true at x". Now it makes sense to say that " 'It rains in Bucharest' is true at (the point corresponding to) the 24^{th} of September 1983"; nevertheless, logicians are in trouble when one wants to take into account a sentence like: "It rained in Bucharest on the 23^{rd} of September 1983". To put it in more general terms: is the relationship W (A is the case at x, $x+1$) (or, even more generally W (A(x), f(x))) well defined? The puzzle is very old and it originates in Aristotle's sea-battle queries.

A very interesting answer is supplied within the frame of bi-dimensional semantics. For example, Segerberg's analysis of "That 'A is true at x' is true at $x+1$" is something like W(A, $x+1$, x) — that is, A is evaluated at two points: at a point x with respect to a point $x+1$. Now here are two problems: the first is that the sentence to be evaluated at the two points is "It rains in Bucharest" and it states a non-modal fact, while "It rained in Bucharest on the 23^{rd} of September 1983" states a modal fact [1]. A possible world, the logician belives, is nothing but a maximal agreggate of non-modal facts [2]. The second problem is that the evaluation of a sentence is to be done at a *pair* of points, or, equivalently, at *a point endowed with a structure* [4, p. 79]. Therefore, the logician meets a dilemma: either a fact like "A is the case at x" is a fact in $x+1$ and therefore he accepts modal facts, or it is not, but in that case A has to be evaluated at points with a more complex structure than it is usually assumed. A choice is needed: Prior, Kamp, van Fraassen, Segerberg and others have chosen the second alternative. As opposed to them, I shall explore the first one.

I. My approach starts with Strawson: he argued that, on different occassions, the same sentence could be used to make different statements. Those statements could be true or false, according to facts. Some authors use to attach truth-values not to statements, but to the non-linguistic entities they express, i.e. propositions. Their standpoint is thus that in

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[1] A seemingly much more significant context for the philosopher's taste is due to A. Plantinga's analysis of essences [2]. He defines world-indexed properties and argues that they are necessary and therefore essential properties of the individuals. For example, being a distinguished philosopher is a contingent property of Quine, for there are worlds in which "Quine is a distinguished philosopher" lacks truth. However, says Plantinga, that actually Quine is a distinguished philosopher is true in all the possible worlds. Therefore, being actually a distinguished philosopher is a necessary property of Quine, thou there it is contingent that in the actual world Quine is a distinguished philosopher. To put it in a somewhat more formal manner: if W(a is P, w), then for every w′ W(a is P-in-w, w′).

The argument is easily formalised in local semantics: if w $\models$ w′ P(a), then for all w″, w $\models$ w″w′P (a). As it will be seen from below, the argument assumes a S5-structure, that is, that for all worlds w_1, w_2, w_1 (w-inner) reflects w_2.

[2] This statement is an essential part of the actualist doctrines about possible worlds. I hope my approach will prove this claim to be wholly ungrounded.

different contexts a sentence could express different propositions (and that is the case when it contains indexical expressions). Then the facts determine if the proposition expressed is true or false. Propositions are usually identified with sets of possible worlds : a proposition is true at a world w if w is a member of the proposition.

B. van Fraassen [5, p. 76] argued that this intuition could be abstracted as follows : each world is equipped with a certain context ; second, each world contains facts. Let A be a sentence : then, every world determines first what is the proposition A does express at it and second whether that proposition is true or not. Of course, given that $/A/_w$ is the proposition corresponding to sentence A by (the context contained in) the world w, then it makes sense to ask if $w' \in /A/_w$, i.e. if the proposition expressed by A at w is true at w′.

And yet van Fraassen's analysis is a bit more complex than it looks like. "The sort of model I will consider here is to be conceived as follows. We have a certain speaker (let it be me). In each world, this speaker is equipped with a certain context (...)" [p. 76]. The sentence "I am here" is true in every world, if, of course, we assume that every context specifies (besides the speaker) a time of utterance and a place of utterance. Furthermore, it "cannot be false, for if it expresses a proposition at all (...), then it expresses a true proposition" [p. 77]. Van Fraassen thought that we could abstract from this complicated situation to bi-dimensional semantics.

I believe that a somehow more adequate model is the following : we have a world and a speaker (let it be me) who is a part of the world and is the source of the contexts. Thus, the world w contains both the speaker (a certain kind of super-context) and facts. That every (other) world is equipped with a context and contains facts are among the *facts* of w. So I take the assertion

Sentence A is true. (1)

and analyse it as :

A speaker *k* defines the occasion *s* on which A is used to make the statement that *p* which is made true by the facts in the world of occasion *s* (2).

It should be noted that the present approach is not reducible to three- (or *n*-) dimensional semantics. A better analogy is perhaps with Kant, if we identify the entity we did call "speaker" with a certain set of conditions of the possibility of the worlds. That is why the possible worlds semantics to be constructed below I labelled transcendental semantics. The speaker's job in (2) is best describable in Kant's own words : "It must be possible for the 'I think' to accompany all my representations ; for otherwise something would be represented to me which could not be thought at all, and that is equivalent to saying that the representation would be impossible, or at least would be nothing to me" (*Critique of Pure Reason*, B 131—132).

A transcendental semantics should work out satisfactory solutions for at least two subjects : the first is that it would be able to define within a possible world tentatively all the semantic relationships we need and, first of all, to construct every possible world as part of any other world. It is this sense in which I called it *local* semantics. (According to local semantics, at every model *m* a world plays a transcendental job ; that is

Let R be one of the relations just defined: then *m* defines an S4-structure if R is transitive; a B-structure if R is symmetrical and an S5-structure if both [5].

III. *Application: van Fraassen's bi-dimensional operators* (see [5])

D2.1. $w \models w'w''WA$ iff $w \models w''A$.
D2.2. $w \models w'w''\boxtimes A$ iff $w \models w'(\forall w''')\, w'''A$.
D2.3. $w \models w'w''\boxdot A$ iff $w \models w''\, (\forall w''')w'''A$.
D2.4. $w \models w'w''\Box A$ iff $w \models (\forall w''')\, (\forall w'''')\, w'''w''''A$.
D2.5. $w \models w'w''\, (A \rightarrow B)$ iff $w \models (\forall w''')\, (w''w'''A \supset w'w'''B)$.

Suppose for simplicity that w' is w. Then we have:

D2.2'. $w \models w'\boxtimes A$ iff $w \models (\forall w'')w''A$.
D2.3'. $w \models w'\boxdot A$ iff $w \models w'(\forall w'')w''A$.

According to D2.3', a w-inner necessity is defined. D2.2' states that w' is able, somehow, to mirror other worlds. $\boxtimes A$ is the case at w' iff A is the case at all w''. Thus, van Fraassen's approach assumes that every world adequately reflects (partially, at least) every other world; that supposition is, however, questionable. Now it is possible to refine van Fraassen's analysis and reduce his outer operators to inner ones. Here, are, for example, two world-indexed operators:

D2.1.1. $w \models w'W_{w''}A$ iff $w \models w'w''A$

(consequently, it is possible to show that two statements' being materially identical is world-relative).

D2.2.1. $w \models w'\boxtimes_{w''}A$ iff $w \models w'w''(\forall w''')w'''A$.

Unlike D2.2., the definition of $\boxtimes_{w''}$ only assumes that w' mirrors w''.

Note. Van Fraassen identifies a proposition with a set of pairs of worlds. As far as local semantics is concerned, a proposition is not identifiable with a set of *n*-tuples of possible worlds. And if one wants to think of it in set-theoretical terms, a proposition could be identified with a class, not a set, equipped with a highly sophisticated structure.

Consider the sentences:

It is raining. (8)
It is raining now. (9)

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[5] It is important to note that relations R are not definable as expressions in WFFM, since quantification over statements is not allowed. However, it is interesting to add R's as primitive symbols, together with axiom-schemas like

If $w \models R(w', w'')$ and $w \models w''A$, then $w \models w'w''A$(3). The formal connexions with the identity relation in PL are easily seen. In the same manner, define the identity of two statements:

$$A = B \text{ iff : } wS \models A \text{ iff } wS \models B \text{ for all } S.$$

If " = " is chosen as a primitive symbol, then

If $w \models A = B$ then $w \models w'A \equiv w'B$ for all w' (4)

and also

If $w \models A = B$ then $w \models A \equiv B$ (5)

are axioms. The consequent of (4) defines what philosophers usually called synonymy, while the consequent of (5) was supposed to define equireferentiality. But identity is a much stronger concept. Indeed, the following hold too:

If $w \models A = B$ then $w \models (\forall w'')\, w''\, (w'A \equiv w'B)$. (6)

and

If $w \models (\forall w'')\, w''\, (w'A \equiv w'B)$ then $w \models w'A \equiv w'B$. (7)

why local semantics could be used to construct a Kantian-type semantics. On the other hand, in so far as I agree with the belief that there are *modal facts* (at every possible world), local semantics is not Kantian. Perhaps the label : *pseudokantian semantics* is best suited).

The second is that it has to provide an analysis of the transcendental arguments. The problem will not be taken into account in the present article ; however, I think it is worth mentioning that the main step towards its solution would be a certain modelling of the mechanisms of autoreference. Note, in this respect, that the possibility that a possible world refers to itself and that this aspect is irreducible is one of the most important features of the calculus to be presented below.

II. Let p, q, r ... be statements. Let me read the expression : wp as "It is the case in w that p" ; or :"p is the case in w" ; or even : "p is true at w". Then ww'p means : "That p is the case at w' is the case at w"[3]. Thus, I assume that a world w contains modal facts : it contains facts concerning what takes place in another world. Let quantification over possible worlds be allowed. Now every modal logician will be ready to equal w (Ew') w'p and w $\Diamond$p : it is the case at w that there is a world w' such that p is the case at w' equals : that p is possible is the case at w.

According to standard modal semantics, w $\Diamond$ p is nothing but a rewriting of $W(\Diamond p, w) = 1$ (if W is a value-assignement function on a certain frame). However, it seems to me that there are in this respect two main differences between the standard account and local semantics : first, W(w'p, w) is not defined on the standard account ; second, within the frame of local semantics it is possible to define "*reflection relations*" between different possible worlds.

The set WFFM of all statements is defined recursively : all atomic statements are in WFFM ; if A and B are in WFFM, then wA, (Ew') w'A, AvB are in WFFM.

A model *m* is a triple (W, w, $\models$), where W is a nonempty set of possible worlds, $w \in W$ and $\models$ is defined as follows :

Let S be a sequence $w_1 w_2 \ldots w_n$ of members of W or the empty sequence and let wS $\models$ A be an abbreviation for w $\models$ SA.

a) wS $\models$ p or wS $\models$ —p for every atomic statement p.

b) wS $\models$ p iff wS $\not\models$ —p for every atomic statement p.

c) wS $\models$ AvB iff wS $\models$ A or wS $\models$ B.

d) wS $\models$ (Ew') w'A iff wS $\models$ w''A for some w''.

(all the logical conectives and the universal quantifier are defined in terms of "—", "v", "E").

e) wS $\models$ w'A iff wS $\models$ w'w'A for every world w' and statement A.

That is the normality condition ; the non-normal systems, which are not considered here, are very important too.

f) wS $\models$ (Ew') w'A iff w $\models$ (Ew') Sw'A.

((f) states the rigidity of world-variables).

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[3] If we read ww'p as : that p is true at w' is true at w, then it is assumed that even metastatements are world-dependent. Possible worlds are taken seriously !

Note that it is possible to move from $w \models A$ to $w \models wA$, but never to $\models wA$; w could not loose its supervising job.

g) $w \models A$ iff $w \models wA$ for all A.
(the locality condition).

An expression A is true at the model $m = (W, w, \models)$ iff $w \models A$. An expression A is valid iff it is true at every model m. Let M be the set of all valid expressions.

Theorem : (i) M is consistent;
(ii) M is undecidable.

I shall sketch the proof of (ii) : let WFF be the set of all well-formed formulas of the predicate logic (PL). Define a map f from WFF to WFFM as follows: if a is an individual constant, then $f(a) \in W$; if x is an individual variable, then $f(x)$ is a world-variable; if P is a predicate letter, then f(P) is an atomic statement in WFFM; $f(P(a_1, a_2 \ldots a_n))$ is $f(a_1)f(a_2) \ldots f(a_n)f(P)$; $f(-X)$ is $-f(X)$; $f(XvY)$ is $f(X)vf(Y)$; $f((Ex_i)P(a_1, \ldots a_{i-1}, x_i, a_{i+1} \ldots a_n)))$ is $f(a_1) \ldots f(a_{i-1})(Ew_i)w_i f(a_{i+1}) \ldots f(a_n) f(P)$. It is easily seen that if X is a thesis of PL, then $f(X) \in M$. But PL is undecidable and therefore M is undecidable. Note also that M is stronger than PL, for f is not onto [4].

Now I start to define the most important relationships describable in the frame of local semantics : *reflection relations;* more specifically, I define *adequate* reflections, but for the sake of simplicity they are referred to by that term.

D1. 1. outer reflection : w′ outer reflects w″ ($R^o(w', w'')$)

$R^o(w', w'') =$ df. if $w' \models w''A$, then $w'' \models A$ for all A.

Note that D1. 1. defines a relation between two models. Definitions D1. 2—D1. 4 aim to eliminate outer reflections in favour of inner ones, so that all the reflection relations among possible worlds would be describable within a single possible world. Possible worlds themselves are viewed as nothing but parts of a certain world; nevertheless, every world is viewed, in the same time, as part of any other world, for indeed it is reflected (adequately or not) by any other possible world.

D1. 2. inner reflection : w′ inner reflects w″ ($R^i(w', w'')$)
$R^i(w', w'') =$ df. if $w' \models w''A$, then $w'' \models w'w''A$ for all A.
D1. 3. outer wS-reflection : w′ outer wS-reflects w″ ($R^{wSo}(w', w'')$)
$R^{wSo}(w', w'') =$ df. if $wS \models w''A$, then $wS \models w'w''A$ for all A.
D1. 4. inner wS-reflection : w′ inner wS-reflects w″ ($R^{wSi}(w', w'')$)
$R^{wSi}(w', w'') =$ df. if $wS \models w'w''A$, then $wS \models w''w'w''A$ for all A.

When S is empty and w′ is w, D1. 4 requires that the picture of w″ in w is the same with the picture in w″ of the reflection of w″ in w (with the picture in w″ of its reflection in w).

Note. The reflection of a world w in another world w′ is a *world* w″ : if the reflection is adequate, then w″ is w and if it is not adequate, then $w \neq w''$ (see also for this subject section IV).

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[4] There are also certain connexions between M and arithmetics, set theory and the theory of categories, but it is not possible to discuss them here. Rescher and Garson's topological logic shares some important features, but not many, with M (see [3]).

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Consider the sentences:

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Their formal counterparts in the frame of local semantics are $w \models p$ (8′) and $w \models wp$ (9′). According to the definition of a model (clause (g)) $w \models p$ iff $w \models wp$ and therefore

$$w \models p \equiv wp.$$

It is raining iff it is raining now. However, we did not meet a case of synonymy, for the two sentences (p and wp) might have a quite different logical behaviour at another possible world w′ (as it is viewed from w).

Yesterday it was the case that it would rain. (10)

Yesterday it was the case that it would rain now. (11)

Their formal counterparts are, respectively, $w \models w'(Ew'')w''p$ (10′) and $w \models w'wp$ (11′). (11′) entails (10′), but the converse does not hold. (In an Aristotelian system, i.e. in one in which w′ (corresponding to "yesterday") is not supposed to reflect adequately (say, w-outer) world w (corresponding to "now"), there is no formal connexion between (11′) and (9′)).

Note. The formal counterpart of (11) is (11′) and not $w'' \models w'wp$ (11″)! "Now" is in the present context a ***rigid term:*** it preserves the reference it gained at w (the "actual" world) in all the possible worlds it supervises; the mechanisms by which this job is done are local (transcendental) ones.

IV. *A local theory of rigidity. A sketch*

Let us move to predicate logic. We have two new quantifiers "∃" and "( )" for individual variables. Now I want to argue in favour of:

$$wS \models S'(\exists x)A(x) \text{ iff } wS \models (\exists x)S'A(x) \text{ for all } S,S'. \qquad (12)$$

(the rigidity of individual variables). Let a be an individual term and let $P(a)$ be a statement (P can be an n-ary predicate, but we are here interested only in a). Suppose there is another individual term b so that $w \models w'(P(a) \equiv P(b))$ is true for every P. I shall say that a and b are w′-equireferential ($a =_{ow'} b$); if w′ is w, say that a and b are equireferential. If the above expression holds for all w′, say that a and b have the same sense (or: they are 1-synonymous). If $w \models S(P(a) \equiv P(b))$ holds for all P and all S with length $n' \leqslant n$ (i.e. of the form $w_1 w_2 \ldots w_{n'}$ with $n' \leqslant n$), say that a and b are n-synonymous ($a =_n b$). If n-synonymy holds for all n, then a and b are synonymous ($a = b$). If these synonymy relations are added as primitive symbols (see also note (5) above), then for instance, the following holds:

If $w \models a =_0 b$ then $w \models P(a) \equiv P(b)$ (13)

However, $w \models a =_0 b$ does not entail $w \models w'P(a) \equiv w'P(b)$, while $w \models a =_1 b$ entails $w \models w'P(a) \equiv w'P(b)$. Of course, those are relations logicians know so well, and $a =_n b$'s generalize them.

Suppose that a and b are w′-equireferential ($a =_{ow'} b$). Now it is possible that

$$w \models w'(P(a) \,.\, P(b)).\ w'(P(a) \,.\, -P(b)). \qquad (14)$$

We have at hand two alternative readings of (14): (i) $w'P(a)$ means that: what "a" refers to at w′ is a P; (ii) $w'P(a)$ means that: what at w "a"

refers to is in w′ a P. *n*-synonymy relations will not help us [6]. For if we say that at w″ "*a*" refers to *X*, while at w′ "*a*" refers to *X*′, then $a_{w'} \neq_1 a_{w''}$[7]. But we don't want to show that; we want to show that "*a*" *cross-refers.*

By the locality principle, $w \models w'P(a)$ iff $w \models ww'P(a)$. What does it mean ? It means either that : (i′) from the standpoint of w (the *actual world)* what at w′ "*a*" refers to is a P, or that (ii′) : from the standpoint of the actual world w what "*a*" refers to in it is in w′ a P (or, in other words : from the standpoint of the actual world : this individual *a* is in w′ a P).

I believe that the first reading is unintelligible, or, at least, mistaken. Let me restate (i′) : *this a* is such that what "*a*" refers to in w′ is in w′ a P, which equals, by the locality principle : what "*a*" refers to in w′ is a P (in w′). Now *a* is the individual that actually is referred to by "*a*". But in (i′)—(ii′) it is assumed that there are some connexions between the individual referred to by "*a*" in the actual world and the individual referred to by "*a*" in w′. It raises the so-called problem of cross-world identification ; but I don't think it is a problem at all : it was made up by a bad philosopher who did not realise that the supervising role of a transcendental context (*world*, i.e. the actual world) is unescapable. It is not possible to conceive of a family of wholly unconnected worlds, at least in the cases when one wants to define semantical relations that would model the behaviour of many philosophically significant sorts of entities.

On the other hand, suppose my restatement of (i′)—(ii′) is unsuited Then the phrase : "from the standpoint of the actual world" does not make any point. But then it is supposed that $w'P(a)$ conveys a statement both about the language (for it comes into account the relation" "*a*" refers to X") and also about a fact (that at w′ the individual *X*, existing in w′, is a P) — which is itself questionable. Moreover, in this case very difficult problems arise with iterated modalities. For what $w''w'P(a)$ would mean ? Either we think that it means the same thing as $w'P(a)$, or that : " "*a*" refers to X_1 in w″ and "*X*" refers to X_2 in w′ and X_2 is in w′ a P". The second reading is monstruous, while the first one is much too simplifying (the S5-assumption).

In fact, it is possible to render the job of (i′) in terms of (ii′). Let λ be a description and suppose that $a =_{0w'} \lambda$ ($w \models w'(P(a) \equiv P(\lambda))$ for all P). Assume that $w \models ((\forall w'')w''P(\lambda) \equiv P(\lambda))$ for all P (= that λ is 1-rigid[8]). Then

$w \models w'(P(a) \equiv P(\lambda))\ .\ w''(P(a) \not\equiv P(\lambda))$ reconstructs (i′).

I have discussed some arguments in support of the claim that individual terms are 1-rigid ; in fact the same serve the claim that they refer *n*-rigidly for every *n* and therefore that they are rigid : in $wS \models P(a)$,

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[6] According to standard modal semantics, it is possible to understand $a =_n b$ as : $\Box^n (P(a) \equiv P(b))$ is, for every P, true in the model.

[7] How to define $a_{w'}$? It is that individual constant that rigidly refers to what "*a*" refers at w′. Some authors worked out solutions for that puzzle. A very well-known one is due to Kaplan : he introduced an operator Dthat. Here we need its relativisation to w′, w″, w‴ and so on.

[8] Descriptions are not usually rigid designators. Indeed, suppose $(\exists x)(P'(x).P''(x))$ is true both at w′ and at w″. But it is possible that at w′ a_1 satisfies this description (more exactly, this part of a description) and at w″ a_2 satisfies it. Now it is apparent that $a_1 \neq_n a_2$ $(n \geqslant 0)$ does not entail anything impossible.

for every S, "*a*" refers to what it refers at w. We have also seen that those arguments do not hold about descriptions.

One would obviously see here a strong analogy with Kripke [1]. Furthermore, some other crucial features of Kripke's "picture" of referring are vindicated by this *local* approach [9]. For all that, there are significant differences. Kripke is right, I guess, that possible worlds should not be identified with the descriptive conditions we associate with them; but he believes that the actual world fulfils the job it does just because all the other worlds are *stipulated* by someone who habitates in it. The actual world is not alike the other worlds : it seems to have a much more complicated structure than those. It contains them and they are dependent upon it. They are stipulated, conceived, while the actual world *is* by itself.

Consequently, the need raised that some statements would be *contingent* (for the facts in the actual world are not necessary) and yet *a priori* (for, according to Kripke's picture of referring, they necessarily hold in all worlds).

Unlike Kripke, I think *there are modal facts :* it is an objective fact about the actual world (and also about any world) that it does furnish the conditions of the possibility of all facts being true (or false) at any other world. We need not to argue for contingent *a priori* truths, but for *contingent 1-necessary facts* (facts that are 1-necessary but, e.g. 2-contingent; this is the case when the following holds for p :

$$w \models ((\forall w')w'p).((Ew')(Ew'')w'w''-p).$$

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4 K. Segerberg, *Two-dimensional Modal Logic*, in "The Journal of Philosophical Logic", 2 (1972), pp. 77–96.

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[9] For example, the so very subtle connexions between proper names and descriptions; their treatment within the frame of local semantics needs, however, the employment of different reflection concepts and also of concepts like : world w″ looks from the standpoint of w like w‴, or : the actual world looks from w′ like w″.